MATHEMATICS

for the

ENVIRONMENTAL HEALTH SPECIALIST

Prepared by:
Virginia Department of Health
Using the Original Manuscript from
J. Sergeant Reynolds Community College

CONTENTS

I. BASIC ARITHMETIC/ALGEBRA

A. Decimals/Fractions

DECIMALS are a special way of writing proper fractions that have denominators ending in zero (0). In other words, fractions that can be written as decimal fractions are those whose denominators are 10 or a power of 10. Thus when written as a decimal fraction,

	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$	$\frac{1}{1,000,000}$
Becomes:	.1	.01	.001	.0001	.00001	.000001

The word decimal means relating to the number 10. The period before the digits is the decimal point. In writing decimal fractions, instead of the denominator being written under the numerator with a bar separating the terms, it is indicated by placing at the right of the decimal point indicates the denominator. This may be shown in the following table:

PLACE OF DIGIT	How to Read It	Example
First decimal place	Tenths	.3 is 3/10
Second decimal place	Hundredths	.03 is 3/100
Third decimal place	Thousandths	.003 is 3/1,000
Fourth decimal place	Ten thousandths	.0003 is 3/10,000
Fifth decimal place	Hundred thousandths	.00003 is 3/100,000

Reading Decimals: Read the number after the decimal point as a whole number and give it the name of the last decimal place.

Thus: 0.135 is read as one hundred thirty-five thousandths

4.18 is read as four and eighteen hundreds

or they may be read as follows: 0.135 is point, one-three-five
4.18 is four, point, one-eight

Write the following as decimals:

1. 3/10
2. 5/100
3. $\frac{321}{1000}$
4. 12 - 1/1000
5. 124 - $\frac{3}{10,000}$
6. 18 - 7/10
7. $\frac{300}{1000}$
8. $\frac{145}{100}$
9. $\frac{223}{10}$
10. $\frac{4330}{1000}$

11. Six and ninety-six hundredths

12. One and six tenths

13. Seventy-five hundred-thousandths

14. Twenty-five and fifty hundredths

15. Thirty-four hundredths

16. Fifteen and two tenths

17. Three and four hundredths

18. Three and fifty four hundredths

19. Twenty-seven thousand, sixty hundred thousandths

20. Ninety and nine hundred seventy-eight thousandths

In problems one through ten you converted fractions to decimals by placing the decimal point and the correct number of ciphers before the numerator and eliminating and the denominator. You could do this because all the denominators were 10's or multiple of ten such as 100, 1,00, 10,000, etc. It is not possible, however, to do this in all cases. Hence -

TO CHANGE ANY COMMON FRACTION INTO DECIMALS, DIVIDE THE NUMERATOR BY THE DENOMINATOR AND WRITE THE QUOTIENT IN DECIMAL FORM.

Examples: Change 3/5 to a decimal Change 3/8 to a decimal

Thus $5\overline{)3.0}$ = 0.6 Ans. Thus $8\overline{)3.00}$ = 0.375 Ans.

The following table gives the decimal equivalents of fractions from 1/32 to 1.

FRACTION	DECIMAL	FRACTION	DECIMAL
1/32	.03125	1/2	.5
1/16	.0625	9/16	.5625
1/8	.125	5/8	.625
3/16	.1875	11/16	.6875
1/4	.25	3/4	.75
5/16	.3125	13/16	.8125
3/8	.375	7/8	.875
7/16	.4375	15/16	.9375

TO ADD OR SUBTRACT DECIMALS, place the numbers in a column with the decimal points in a column. Add or subtract as for whole numbers, placing the decimal point in the result in the column of decimal points.

Examples: Find the sum of 2.43, 1.485, 12.01 and 0.74

```
 2.43     or     2.430
 1.485           1.485
 0.3             0.300
12.01           12.010
  .074            .074
------          ------
16.299          16.299
```

Since 1.485 and .074 are three place numbers, we write zeros after 2.43, .3 and 12.02. This does not change the value of the number but helps to avoid errors.

Find the difference between 17.29 and 6.147

```
17.29      or     17.290   Here, as above, we added a zero to the
-6.147            -6.147   17.29 to help avoid making errors.
11.143            11.143
```

Add the following:

1. 0.2 + 0.07 + 0.5 =
2. 2.6 + -22.4 + 0.03 =
3. 22.8 + 5.009 + 613.2 =
4. 0.005 + 5 + 16.2 + 0.96 =
5. 15.4 + 22 + 0.01 + 1.43 =
6. 28.74 + 16.32 =
7. 0.005 + 0.0005 =
8. 1.431 + .562 =
9. 1.0021 + 0.2 =
10. 72.306 + 18.45 + 27.202 =

TO MULTIPLY DECIMALS, proceed as in the multiplication of whole numbers. But in the product, beginning at the right, point off as many places as there are in the multiplier and in the multiplicand.

EXAMPLE: Multiply 3.12 by 0.42

```
  3.12     multiplicand (has two decimal places)
  0.42     multiplier
   624
 1248
1.3104     Since there are a total of four decimal places
           when we add together those in the multiplier and
           the multiplicand, we start at the right and count
           four places; hence we place the decimal point in
           front of the 3.
```

Multiply 0.214 by .303

```
  0.214         There are a total of six places in the multiplier
  0.303         and multiplicand but only five numbers in the
    642         product; therefore we prefix a zero at the left
  6420          end and place our decimal point to the left of it
 .64842         to give our answer the required six decimal
.064842 Ans.    places.  If we needed eight places and the answer
                came out to five places we would add three zeros
                and place the decimal point to the left of them.
```

TO DIVIDE A DECIMAL BY ANY MULTIPLE OF TEN, move decimal point as many places to the left as there are zeros in the multiplier.

Thus, 42 ÷ 10 = 4.2   15.6 ÷ 100 = .156   61 ÷ 1000 = .061

Solve the following problems:

1. 18.5 x 4 =
2. 3.9 x 2.4 =
3. 45 x 0.72 =
4. 143 x 0.214 =
5. 0.56 x 0.74 =
6. 0.224 x 0.302 =
7. 9.06 x 0.045 =
8. 0.008 x 751.1 =
9. 8.7 x 10 =
10. 0.0069 x 10 =
11. 0.0454 x 100 =
12. 492.569 ÷ 1,000 =

13. 534.79 ÷ 100 =
14. 24.9653 ÷ 1,000 =
15. 0.0716 ÷ 1,000 =
16. 0.038649 ÷ 100,000 =

LAW OF DIVISION - A quotient is not changed when the dividend and the divisor are both multiplied by the same number.

Example: 7.2 ÷ .9 =
7.2 x 10 = 72
0.9 x 10 = 9
72 ÷ 9 = 8

To DIVIDE A DECIMAL BY A WHOLE NUMBER, proceed as with whole numbers, but place the decimal point in the quotient directly above the decimal point in the dividend.

Example: 20.46 ÷ 66

```
       .31  Ans.
   66)20.46
      198
       66
       66
        0
```

How many yards are there in 165.6 inches?

```
      4.6  Ans.
  36)165.6
     144
      216
      216
        0
```

TO DIVIDE A DECIMAL BY A DECIMAL, move the decimal point of the divisor to the right until it becomes a whole number. Next move the decimal point of the dividend the same number of places to the right, adding zeros if necessary. Then proceed to divide as above.

EXAMPLE: 131.88 ÷ 4.2

```
     31.4 Ans.
 4.2)131.88
     126
       58
       42
      168
      168
        0
```

Division of a decimal by a decimal is simplified if the divisor is made a whole number. In this case the divisor 4.2 was made a whole number by moving the decimal point one place to the right; therefore, we also moved the point in the dividend one point to the right. Then placing the decimal point in the quotient directly above the decimal point in the dividend, we proceed as for division of whole numbers.

Examples (Continued)

6168 ÷ 0.05

```
  123360  Ans.
5)616800
  5
  11
  10
   16
   15
    18
    15
     30
     30
      0
```

Moving the decimal point in the divisor two places to the right we get 5, a whole number. Since there is no decimal point in the dividend we add two zeros and get 616800.

61.68 ÷ 0.05

```
  1233.6 Ans.
5)6168.0
  5
  11
  10
   16
   15
    18
    15
     30
     30
      0
```

Again we move the decimal point in the divisor two spaces to the right giving us the whole number 5. We then move the decimal point in the dividend two places to the right giving us the figure 6168. We then divide as with a whole number.

Divide the following, carrying the quotient only to three decimal places:

1. 613.2 ÷ 0.2 =
2. 9.171 ÷ 9 =
3. 0.6117 ÷ 3 =
4. 8.2 ÷ 8 =
5. 413.2 ÷ 4 =
6. 7189 ÷ 0.7 =
7. 5145 ÷ 0.005 =
8. 8200 ÷ 2.5 =
9. 6168 ÷ 0.06 =
10. 5160 ÷ 0.5 =
11. 9.117 ÷ 0.009 =
12. 0.815 ÷ 0.002 =
13. 610.8 ÷ 0.06 =
14. 81.92 ÷ 0.005 =
15. 0.081 ÷ 0.0022 =
16. 2.83 ÷ 0.007 =
17. 0.987 ÷ 21 =
18. 0.2546 ÷ 0.38 =

Problems in decimals

1. If the gas rate in a certain town is $0.45 per 1,000 cubic feet, how much is it for one cubic foot?

2. If there are 144 pens in a gross costing $0.75, what is the cost of 1-1/2 dozen pens?

3. Ice weighs .92 as much as water and 1 cubic foot of water 62.4 lbs. What is the weight of a block of ice three cubic feet in size?

4. A man owns two pieces of property. In one there are 2.25 acres and in the other one 1.125 acres. If he sold the entire property

for \$2700, how much would he get per acre?

B. Per Cent

The phrase per cent means "by the hundred." So, we use the percent symbol, %, to indicate division by 100.

For example, when we say that 10% of the students have blue eyes, we mean that 10 out of every 100 students have blue eyes. We can also write 10% as a fraction 10/100, or as a decimal 0.10.

Rule for changing a percent into a decimal fraction.

Examples:
21% = 0.21
7% = 0.07
175% = 1.75
1/4% = 0.25% = 0.0025
7.5% = 0.075

Rule for changing a decimal fraction into percent.

Examples:
0.6 = 60%
1 = 100%
1 1/4 = 1.25 = 125%
0.0075 = 0.75%
.060 = 6%

Problems:

1. How many pounds of salt are there in 25 pounds of a 10% salt brine?

   25 x 10% = 25 x 0.10 = 2.5 pounds of salt

2. A soap is advertised to be 99 44/100 pure. How many pounds of impurities could there be in 1 ton (2000 pounds) of this soap.

C. Ratios and Proportions

A ratio of a number x to a number y ($y \neq 0$) is the quotient x/y. Thus, a fraction is a ratio.

Example: The ratio of the density of an object to the density of water is known as the specific gravity of the object. If the density of gold is 1200 lb/ft^3 and the density of water is 62.4 lb/ft^3, what is the specific gravity of gold?

$$\frac{1200 \text{ lb/ft}^3}{62.4 \text{ lb/ft}^3} = 19.23 \text{ (Specific gravity of gold)}$$

A proportion is a statement of equality between two ratios.

Example: If 3 lbs. of salt are added to 10 gallons of water to make a solution of a given strength, how many pounds would be added to

129 gallons to make a solution of equal concentration?

$$\frac{3 \text{ lbs}}{10 \text{ gal}} = \frac{x \text{ lbs}}{129 \text{ gal}}$$

Multiply diagonally across,

$$X(10 \text{ Gal}) = (3 \text{ lbs})\ (129 \text{ Gal})$$

$$X = \frac{(3 \text{ lbs})\ (129 \text{ Gal})}{(10 \text{ Gal})}$$

$$X = 38.7 \text{ lbs.}$$

Although proportions are usually not difficult to solve, some care must be taken when using them. Some varying quantities are inversely proportional to each other. Their products, not their ratios, are constant.

Example: If three men can do a certain job in 20 hours, how long would it take five men to do the same job?

NOTE: This problem is inversely proportional. If this fact were not noticed, many would solve it by direct proportion.

$$\frac{3 \text{ men}}{20 \text{ hours}} = \frac{5 \text{ men}}{x \text{ hours}}$$

$$x(3 \text{ men}) = (20 \text{ hours})\ (5 \text{ men})$$

$$x = \frac{(20 \text{ hours})\ (5 \text{ men})}{3 \text{ men}}$$

$x = 33\ 1/3$ hours [WRONG]

The solution is wrong since increasing the manpower should decrease the time required for the job. The problem is therefore inversely proportional and the products of the varying quantities should be equated.

$$(3 \text{ men})\ (20 \text{ hours}) = (5 \text{ men})\ (x \text{ hours})$$

$$x = \frac{(3 \text{ men})\ (20 \text{ hours})}{5 \text{ men}}$$

$x = 12$ hours

D. Isolation of a Desired Variable

The following transformations or operations can be made on equations and the equality will be maintained:

1. The same value may be added to or subtracted from both sides of the equation.

   Examples: (a) $x = 4$, $x+3 = 4+3$, or $x-2 = 4-2$ (b) $x = z+2$, $x+A = z+2+A$, or $x-B = z+2-B$

2. The same value may be multiplied or divided by both sides of an equation.

   Examples: (a) $x = 4$, $3x = (3)*(4) = 12$, or $\frac{x}{2} = \frac{4}{2} = 2$ (b) $Y = z+2$, $AY = Az+2A$, or $\frac{Y}{B} = \frac{z}{B} + \frac{2}{B} = \frac{z+2}{B}$

3. Both sides of an equation may be raised to the same power.

   Examples: (a) $x = 4$, $x^2 = 4^2$, $x^2 = 16$ (b) $Y = z+2$, $Y^2 = (z+2)^2$, $Y^2 = z^2+4z+4$

4. The same root of each side may be taken.

   Examples: (a) $x^2 = 9$, $\sqrt{x^2} = \sqrt{9}$, $x = 3$ (b) $Y^2 = z+2$, $\sqrt{Y^2} = \sqrt{z+2}$, $Y = \sqrt{z+2}$

In order to isolate the desired variable from an equation, one or more of the above transformations will be necessary.

Example: Area of Circle, $A = \pi R^2$
Solve the equation for R.

Rearrange the equation,
$$\pi R^2 = A$$

Divide both sides by π,
$$\frac{\pi R^2}{\pi} = \frac{A}{\pi}$$

Take square root of both sides.
$$\sqrt{R^2} = \frac{A}{\pi}$$

$$R = \frac{A}{\pi} = \frac{A}{\pi}^{1/2} = \frac{A}{\pi}^{0.5}$$

NOTE: The important thing to remember when solving an equation is that the equality must always be maintained, regardless of the transformation.

Problem: (a) Area of Square, $A = L^2$, solve for L.

(b) Area of Triangle, $A = \frac{1}{2}BH$, solve for H.

(c) Volume of Circular Tank, $V = \pi R^2 \times H$, solve for R.

E. Scientific Notation/Logarithms

1. Exponential or Scientific Numbers
   a. General
   Method used to more easily express very large and very small numbers.

Ordinary Number	Exponential Form
1,000,000	10^6
100,000	10^5
10,000	10^4
1,000	10^3
100	10^2
10	10^1
1	10^0
0.1	10^{-1}
0.01	10^{-2}
0.001	10^{-3}
0.0001	10^{-4}
0.00001	10^{-5}
0.000001	10^{-6}

Other Examples:

1. $8,270,000 = 8.27 \times 10^6 = 82.7 \times 10^5 = 827 \times 10^4$ etc.
2. $0.0049 = 4.9 \times 10^{-3}$ same as saying 4.9×0.001 or 49×10^{-4}

b. Addition and Subtraction

1. Add or subtract as the case may be - the numbers involved.

2. But - to do so - the powers of 10 must always be the same.

Examples:

1) Add $6 \times 10^3 + 4 \times 10^3 =$ (1) $6 + 4 = 10$ (2) 10^3
   Answer $= 10 \times 10^3$
   Check: $6 \times 10^3 = 6,000$
   $4 \times 10^3 = 4,000$
   TOTAL $= 10,000$
   and $10 \times 10^3 = 10,000$

2) Add $6 \times 10^3 + 0.4 \times 10^4$

   remember to always make the powers of 10 equal:
   $6 \times 10^3 = 0.6 \times 10^4$
   so $0.6 \times 10^4 + 0.4 \times 10^4 = 1.0 \times 10^4 = 10,000$

   or you can work it the other way:
   $0.4 \times 10^4 = 4.0 \times 10^3$
   so $4.0 \times 10^3 + 6 \times 10^3 = 10 \times 10^3 = 10,000$

3) Add 4.2×10^3 and 6.0×10^{-1}
$4.2 \times 10^3 = 4200 = 42000 \times 10^{-1}$

so 42000×10^{-1}
$+ \; 6 \times 10^{-1}$
ans. $42006 \times 10^{-1} = \underline{4200.6}$

4) Subtract

$6.7 \times 10^{-2} - 4.8 \times 10^{-4}$
$6.7 \times 10^{-2} - .048 \times 10^{-2} = 6.652 \times 10^{-2}$

or

$6.7 \times 10^{-2} = 670 \times 10^{-4}$
$670 \times 10^{-4} - 4.8 \times 10^{-4} = 665.2 \times 10^{-4}$
$= 6.652 \times 10^{-2}$

c. Multiplication

When powers of ten are multiplied, the exponents are added. It is evident that $10^1 \times 10^1 = 10^2$ or 100
$10^2 \times 10^3 = 10^5$ or $100 \times 1000 = 100{,}000 = 10^5$
When numbers are involved, the factors which multiply the powers of ten are multiplied separately.

Example: $(2 \times 10^5)(4 \times 10^6) = (2 \times 4)(10^5 \times 10^6)$
Answer : 8×10^{11}

Examples:

1) $(4 \times 10^{10})(6 \times 10^3)(2 \times 10^5) = 48 \times 10^{18} = 4.8 \times 10^{19}$
2) $(5.2 \times 10^{-3})(2 \times 10^2) = 10.4 \times 10^{-1} = 1.04$
3) $(7 \times 10^6)(7 \times 10^{-6}) = 49$
4) $(4.04 \times 10^{23})(1 \times 10^6) = 4.04 \times 10^{29}$
5) $(3.2 \times 10^{-3})(4.1 \times 10^{-4}) = 13.12 \times 10^{-7} = 1.312 \times 10^{-6})$

d. Division

Same as multiplication except the powers of ten are subtracted - that of the divisor from that of the dividend.

$10^5/10^2 = 100{,}000/100 = 1000$ this is the same as $10^5 \div 10^2 = 10^{(5-2)} = 10^3$

--The factors multiplying the powers
$6 \times 10^8 / 2 \times 10^5 = 3 \times 10^3$

1) $\dfrac{14 \times 10^{16}}{7 \times 10^8} = 2 \times 10^8$

2) $\dfrac{8 \times 10^{-9}}{4 \times 10^{-5}} = 2 \times 10^{-4}$

3) $\dfrac{6 \times 10^{-2}}{3 \times 10^{10}} = 2 \times 10^{-12}$

4) $\frac{7.8 \times 10^{3}}{2 \times 10^{-9}} = 3.9 \times 10^{12}$

5) $\frac{9 \times 10^{32}}{6 \times 10^{-15}} = 1.5 \times 10^{47}$

2. Logarithms

The exponent of the power to which the number 10 must be raised to give a certain number is the logarithm of the number.

Thus: 4 is the log. of 10^{4} (or 10,000)
-4 is the log. of 10^{-4} (or 0.0001)

Number	Exponential Form	Log.
1,000,000	10^{6}	6
100,000	10^{5}	5
10,000	10^{4}	4
1,000	10^{3}	3
100	10^{2}	2
10	10^{1}	1
1	10^{0}	0
0.1	10^{-1}	-1
0.01	10^{-2}	-2
0.001	10^{-3}	-3
0.0001	10^{-4}	-4
0.00001	10^{-5}	-5
0.000001	10^{-6}	-6

a. Multiplication

1. Longhand Method
   100,000 x 0.001 = 100

2. Exponential Method
   $10^{5} \times 10^{-3} = 10^{(5-3)} = 10^{2} = 100$

3. Log. Form (multiplication - add logs)

Log.	100,000 =	5.0000
Log.	0.001 =	-3.0000
Log.	answer	2.0000

The number whose log. is equal to 2.0000 = answer = 100

b. Division

1. Longhand Method
   10,000 ÷ 0.0001 = 100,000,000

2. Exponential Method
   $10^{4} \div 10^{-4} = 10^{4-(-4)} = 10^{8} = 100,000,000$

3. Log. Form (division - subtract logs)

Log.	100,000 =	5.0000
Log.	0.001 =	(-)-3.0000
Log.	answer	8.0000

c. Parts of Logarithms

General
Logs are divided into two parts
a) the characteristic
b) the mantissa

Example: The log of the number 2678 is = 3.7811
Here the characteristic is 3 and the mantissa is 0.7811

The Characteristic

For numbers greater than one the characteristic is positive and equal to the number of figures to the left of the decimal place minus one.

Examples:

Number	Characteristics
2678	3
34	1
678000	5
2	0

Also for numbers less than one the characteristic is negative and one unit greater (in a negative direction) than the number of ciphers before the first significant figure.

Examples:

Number	Characteristics
.01	-2
.00682	-3
.00342	-3
.000012	-5

The Mantissa

This mantissa is always found from a table. The mantissa is always the same for all numbers having the same significant figures arranged in the same order.

Examples:

Number	Characteristics	Mantissa
450	2	.6532
45.0	1	.6532
4.5	0	.6532
0.450	-1	.6532

However the characteristic varies.

Examples of problems worked by using logs:

1. Multiplication (add logs)

   0.8427 x 378.2

   log 0.8427 = -1.9257 - 10.9257 - 11
   log 378.2 2.5777
   log of ans. 13.5034 - 11

   13.5034 - 11 = 2.5034 (also, log of answer) Look up mantissa (0.5034) in a chart and find number 3187. The characteristic is two which tells us the number = 318.7

2. Division (subtract logs)

   694.3 divided by 0.00376
   log 694.3 = 2.8416 = 12.8416 - 10
   log 0.00376 = -3.5752 = 7.5752 - 10

   The subtract the logs:

   $$\begin{array}{r} 12.8416 - 10 \\ \underline{-7.5752 - 10} \\ 5.2664 \end{array}$$

   log of answer = 5.2664 (look up 0.2664 in table and find answer = 184,700 no. of figures given by characteristic

3. Solve the problem:

   $$\frac{873.9 \times 246 \times 0.482 \times 0.00291}{0.732 \times 465 \times 1.9}$$

Solution:	log 873.9	2.9415
	log 246	2.3909
	log 0.482	0.6831 -1
	log 0.00291	0.4639 -3
	log numerator	6.4794 -4 = 2.4694
	log 0.732	0.8645
	log 465	2.6675
	log 1.90	0.2788
	log denominator	3.81808 -1 = 2.8108
	log numerator = 2.4794	3.4764 -1
	minus log denominator = 2.8108	2.8108
	log answer =	0.6686 -1

From table 0.6686 = number 4662
Characteristic (-1) tells us the answer is 0.4662

II. MEASUREMENT

Did you know that all measurement can be expressed in only three units, with which you are familiar and that you use every day? These are length, weight, and time. In this country we use inches, feet, yards, miles, etc., as units for measuring length. We use ounces, pounds, tons, etc., as units for measuring weight. We use seconds, hours, days, etc. as units for measuring time. Try as you may, it is impossible to find a unit of measurement that cannot be broken down into three units. Let us test this out and see for ourselves.

Abbreviations of Units of Measurement

in. = inch	in.^{2} = square inch	in.^{3} = cubic inch
ft. = foot	ft.^{2} = square foot	ft.^{3} = cubic foot
yd. = yard	lb. = pound	cfs. = cubic feet per second
sec. = second	gal. = gallon	MGD. = million gallons per day
hr. = hour		

Table of Measurement Units

Linear Measure

1 foot = 12 inches; 1 yard = 3 feet; 1 mile = 5,280 feet

Square Measure

1 square foot = 144 square inches; 1 square yard = 9 square feet; 1 acre = 43,560 square feet

Cubic Measure

1 cubic foot = 1728 cubic inches; 1 cubic yard = 27 cubic feet;
1 acre foot = 43,560 cubic feet; 1 cubic foot = 7.48 gallons;
1 gallon = 231 cubic inches; 1 gallon = 0.1337 cubic feet;
1 gallon = 3.785 liters; 1 quart = 0.946 liters; 1 liter = 1,000 cubic centimeters

Weight Measure (Water)

1 gallon = 8.34 pounds; 1 cubic foot = 62.4 pounds

Definition of Prefix of Units

mega = 1,000,000	micro = $\frac{1}{1,000,000}$
kilo = 1,000	milli = $\frac{1}{1,000}$
hecto = 100	centi = $\frac{1}{100}$
deka = 10	deci = $\frac{1}{10}$

A. Length

Length is measured in inches, feet, yard, miles. All of these units of measurement are interchangeable, or they can all be broken down in terms of each other. You know that 1 foot = 12 inches and that 1 yard = 3 feet. In the same way 1 foot = 1/3 yard and 1 inch = 1/12 foot. Why is this so? It is so because men have agreed that it is so -- it is a matter of definition. When we say "yard" we know what we are talking about, since we have agreed on what a yard is: it is 3 feet or 36 inches. We have now learned to measure length or to measure how long something is. When we say something is "6 feet long" we are measuring it in one direction or one dimension but suppose we want to measure it in two directions or two dimensions. We just measure the length in each of the directions, and we can convert both of these figures to a single unit which we call "area."

Units of Length

a. United States

Units: inches, feet, yards
Examples: 12 inches (in,") = 1 foot (ft., ')

b. Metric System

Units: millimeter (mm), centimeter (cm), meter (m)
Examples: 10 mm = 1 cm
100 cm = 1 m
1000 mm = 1 m

c. Conversion

Examples: 1 ft. = 30.48 cm
1 ft. = 0.3048 m

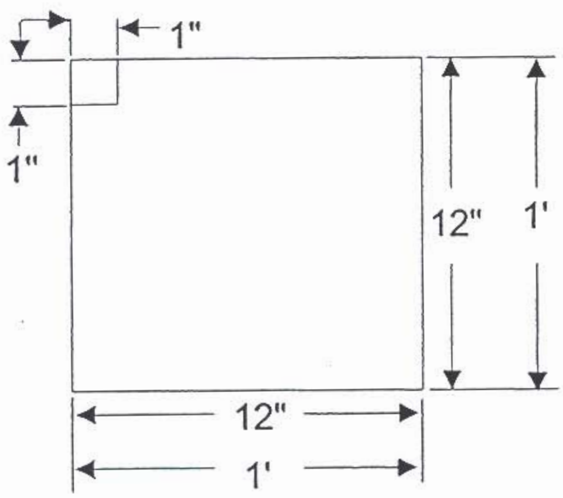
B. Area

Area is the measurement in two directions or two dimensions. When measuring length we used inches, feet, etc., but when we measure area it is expressed in square inches, square feet, square yards, and square miles. The term "square" has a simple meaning in arithmetic -- it means a number multiplied by itself. In other words if the chair seat you are now sitting on is 1 foot long in each direction you are sitting on 1 square foot in area.

Units of Area

a. United States

Units: square inch (sq.in., in.^{2}), square foot (sq.ft., ft.^{2}) square yard (sq.yd., yd.^{2}), acre



1" x 1" = 1 in.^{2}

12" x 12" = 144 sq.in.

12" = 1 ft.

1' x 1' = 144 in.^{2} = 1 ft.^{2}

3' = 1 yard

3' x 3' = 1 sq.yd.

1 acre = 43,560 ft.^{2}

1 acre = 209 ft. x 209 ft.

b. Metric

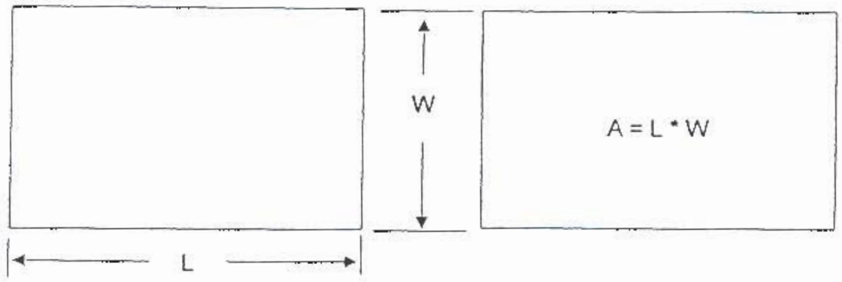
Units: square centimeters (cm.^{2}), square meters (m^2)

Examples: 1 cm x 1 cm = 1 cm^2
1 cm x 5 cm = 5 cm^2

c. Conversion

Examples: 1 m^2 = 10.76 ft.^{2}
1 cm^2 = 0.155 in.^{2}

1. Rectangle: The area of a rectangle is equal to its length (L) multiplied by its width (W).



Example: Find the area of a sedimentation tank if the length is 45 feet and the width is 15 feet.

Area, sq.ft. = Length, ft. x width, ft.
= 45 ft. x 15 ft.
= 675 $ft.^2$
= 675 sq. ft.

Example: The surface area of a sedimentation tank is 1,600 sq.ft. One side measures 20 feet. How long is the other side?

A = L x W
1600 sq.ft. = L ft. x 20 ft.

$$\frac{1600\ ft.^2}{20\ ft.} = \frac{L\ ft. \times \cancel{20\ ft.}}{\cancel{20\ ft.}}$$ Divide both sides of the equation by 20 ft.

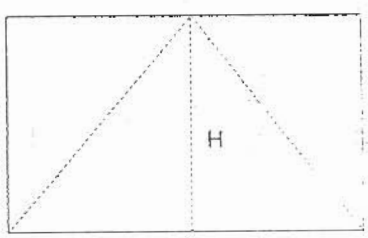
$$\frac{1600\ ft.^2}{20\ ft.} = L\ ft.$$

80 ft. = L ft.

Therefore: the other side of the sedimentation tank is 80 ft. long.

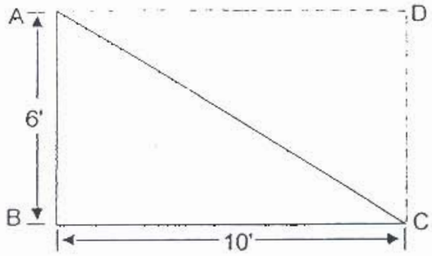
2. Triangle: The area of a triangle is equal to one-half of the base multiplied by the height.

A = (1/2)*B*H



Note: The area of **any** triangle is equal to ½ the area of the rectangle that can be drawn around it. The area of the rectangle is B x H. The area of the triangle is ½B x H.

Example: Find the area of the triangle ABC:



$$\text{Area, sq.ft.} = \tfrac{1}{2}(\text{Base, ft.})(\text{Height, ft.})$$

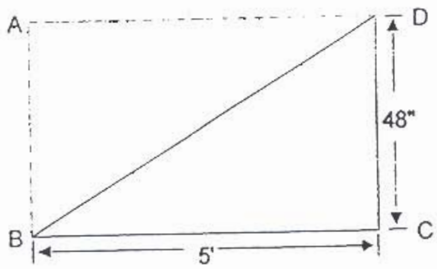
$$= \tfrac{1}{2} \times 10 \text{ ft.} \times 6 \text{ ft.}$$

$$= \tfrac{1}{2} \times 60 \text{ ft.}^2$$

$$= \frac{60 \text{ ft.}^2}{2}$$

$$= 30 \text{ sq.ft.}$$

Example: Find the area of triangle BCD:



Note: It is important to make all the units the same, in this case, it is easier to change inches to feet.

$$48 \text{ in.} = 48 \text{ in.} \times \frac{1 \text{ ft.}}{12 \text{ in.}} = \frac{48 \text{ ft.}}{12} = 4 \text{ ft.}$$

$$\text{Area, sq. ft.} = \tfrac{1}{2}(\text{Base, ft.})(\text{Height, ft.})$$

$$= \tfrac{1}{2} \times 5 \text{ ft.} \times 4 \text{ ft.}$$

$$= \tfrac{1}{2} \times 20 \text{ ft.}^2$$

$$= \frac{20 \text{ ft.}^2}{2}$$

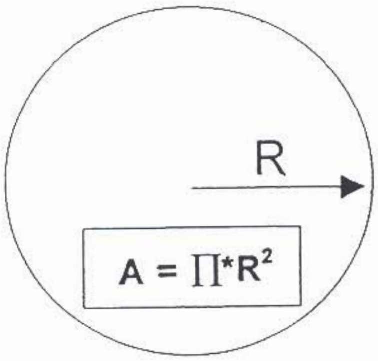
$$= 10 \text{ sq.ft.}$$

3. Circle: The area of a circle is 3.14 multiplied by the radius squared.

The formula for the area of a circle is usually written:

$$A = \pi R^2$$

The Greek letter π (pronounced pie) merely substitutes for the value 3.14.



Example: What is the area of a trickling filter with a 50-foot radius?

$$\begin{aligned} \text{Area, sq.ft.} &= \pi R^2 \\ &= 3.14 \times 50^2 \\ &= 3.14 \times (50 \times 50) \\ &= 3.14 \times 2500 \\ &= 7850 \text{ sq.ft.} \end{aligned}$$

One thing that you must remember is that when working problems of measurement we not only multiply or divide the numbers, but also the terms or units, such as feet, gallons, pounds or whatever they may be. We multiplied feet by feet and got feet^2 or square feet. You have now learned to measure the area of a rectangle or square, but suppose you want to measure the area of an object of a different shape. Well, you just use the formula for that shaped object. The answer is again written in square feet. O.K., we now know how to measure area or to measure something in two directions or two dimensions. But many things have three dimensions--how do we measure these? Simple, we just measure the length in each of the three directions or dimensions, and we can convert all three of these figures to a single unit which we call "volume."

C. Volume

Volume is the measurement in three directions or three dimensions. When measuring length we used inches, feet, etc., and when measuring area we used square inches, square feet, etc., but when we measure volume it is expressed in cubic inches, cubic feet, cubic yards, cubic miles. Again the term "cube" in arithmetic is the source of the expression "cubic unit of length" or "volume." The cube of a number is that number multiplied by itself three times and is written with a 3 above the right side of the number cubed. Thus, $5 \times 5 \times 5 = 5^3 = 125$. In the same manner we can multiply the "units of measurement." Thus, if we multiply 5 feet by 5 feet by 5 feet the answer is 5^3 $feet^3$ or 125 $feet^3$ or 125 cubic feet.

Units of Volume

A. United States

Units: gallon (gal.), cubic feet (cu.ft., $ft.^3$)
quarts (qts.)

B. Metric

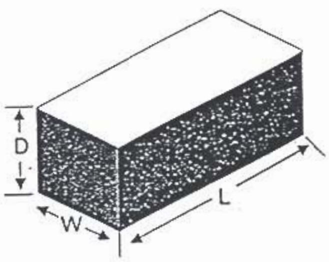
Units: milliliters (ml., cc), liters (l)

C. Conversion

1 gal. = 8.34 pounds
1 cu.ft. = 7.48 gallons
1 cu.ft. = 62.4 lbs. (water)
1 liter = 1,000 ml.
1 liter = 1.056 quarts

1. Rectangular Tank or Box

Volumes are measured in three dimensions or in cubic units. To calculate the volume of a rectangular tank, multiply the length by the width by the height (or depth).



$$V = L * W * D$$

Example: A rectangular settling tank is 60 feet long, 15 feet wide, and 8 feet deep. Calculate the volume in cubic feet.

Volume, cu.ft. = Length, ft. x width, ft. x depth, ft.
= 60 ft. x 15 ft. x 8 ft.
= 900 sq.ft. x 8 ft.
= 7200 cu.ft.

Note: Conversion of Feet and Inches to Feet. In many sanitation problems it is important to know the volume of a cylinder or the area of an object. If the yard stick, carpenter's rule etc., are used to measure the size of the object, the dimensions are obtained in feet and inches. It is impossible to determine the volume or area of an object by using the dimensions as given in feet and inches. They cannot be expressed that way. The dimensions must be converted to either feet or inches and then the area of volume can be determined. As many of our units and relations are in feet, cubic feet etc. (i.e., 1 cu.ft. of water weighs 62.4 pounds) the measured dimensions must be converted to feet and decimals of feet in order to be used in mathematical equations (2 apples + 2 pears do not equal 4 apple-pears).

Since there are 12 inches in a foot, any dimension expressed in inches can be changed to a decimal of a foot by dividing by 12.

Example: Express 7 inches in feet.

Divide 7 by 12 as follows

7/12 = 0.58 ft. (In our work the result to 2 decimal places is sufficiently accurate)

Example: Convert the measurement 5 ft. 5 inches to feet and decimals of a foot.

First set up the fraction 5 5/12. Divide 5 by 12 and add to 5.0 as follows: 5 + 5/12 = 5.42 ft.

Example: What is the volume in cubic feet of a rectangular tank 3 ft. 9 in. wide, 4 ft. 3 in. long and 2 ft. deep?

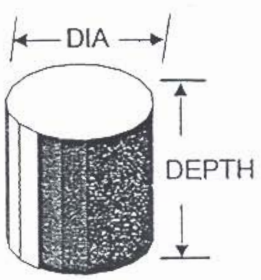
First convert dimensions to feet.

3 ft. 9 in. = 3 9/12 = 3.75
4 ft. 3 in. = 4 3/12 = 4.25

Volume = 3.75 x 4.25 x 2.0 = 31.88 feet^3

2. Cylindrical Tank

The volume of a cylindrical tank is equal to the area of the base multiplied by the height (or depth).



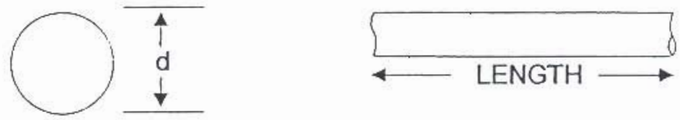
$$V = \prod R^2 * \text{DEPTH}$$

Example: A primary clarifier has a diameter of 90 ft. and a depth of 12 ft. Calculate the volume.

$$\begin{aligned}\text{Volume, cu.ft.} &= \pi R^2 \times \text{depth}\\ &= 3.14 \times 45^2 \times 12\\ &= 3.14 \times 2025 \times 12\\ &= 3.14 \times 24{,}300\\ &= 76{,}302 \text{ cu.ft.}\end{aligned}$$

3. Of a Pipe (Cylinder)

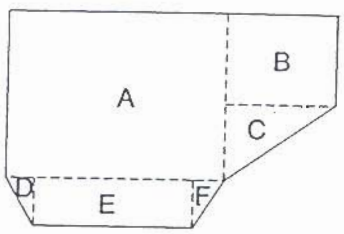
Volume = area of end times the length



area = $\frac{\pi d^2}{4}$ or πr^2 volume = $\frac{\pi d^2}{4}$ x (length)

volume = πr^2 x (length)

4. Of an Irregular Shaped Tank



Square footage of A, B, C, D, E, & F multiplied by the width of the tank = volume ($ft.^3$)

Now, in addition to measuring volume in cubic inches or cubic feet we frequently want to measure it in gallons. How do we do that? You remember that the units of measurement are as they are because people have agreed on them. The same applies to the definition of a U.S. gallon. We have agreed that one gallon is equal to 231 cubic inches. One cubic foot is about 7.48 gallons. If the volume of a tank is 1600 cubic feet or 1600 $feet^3$, to find its volume in gallons, we multiply the volume in cubic feet by the number of gallons in one cubic foot. Therefore,

1600 cubic feet x 7.48 gallons per cubic foot =

$$1600\ \cancel{ft.}^3 \times \frac{7.48}{\cancel{ft.}^3}\ \text{gallons} = 11{,}968\ \text{gallons}$$

Note that when writing gallons per cubic foot in an equation it is written $\frac{gals.}{ft.^3}$

This will apply to any expression with the word "per" in it that we use here. Other expressions written in the same way are pounds per square inch = $\frac{lbs.}{in.^2}$; cubic feet per second = $\frac{ft.^3}{sec.}$. Note also that in the problem above we divided the units as well as multiplied the numbers before these units. $\frac{\cancel{ft.}^3}{\cancel{ft.}^3}$ = 1 and thus cancel each other, the only remaining unit as gallons.

D. Weight

Units of Weight

A. United States

Units: pound (lb.), ounces (oz.)
Example: 1 lb. = 16 oz.

B. Metric

Units: kilogram (kg), gram (gm), milligram (mg)
Examples: 1 kg = 1,000 gms
1 gm = 1,000 mgs
1 kg = 1,000,000 mgs

C. Conversion

Examples: 1 kg = 2.2 lbs.

E. Weight-Volume Relations

A gallon of water weighs 8.34 lbs., and a cubic foot of water weights 62.4 lbs. If these two weights are divided, it is possible to determine the number of gallons in a cubic foot.

$$\frac{62.4 \text{ pounds/cu.ft.}}{8.34 \text{ pounds/gal.}} = 7.48 \text{ gal/cu.ft. or } 7.50 \text{ gal/cu.ft.}$$

1. Example: Calculate the number of gallons in 500 cu.ft.

   500 cu.ft. x 7.48 gal/cu.ft. = 3740 gallons

2. Example: What is the weight of three cubic feet of water?

   62.4 lb./cu.ft. x 3 cu.ft. = 187.2 lbs.

3. Example: The net weight of a tank of water is 750 lbs. How many gallons does it contain?

$$\frac{750 \cancel{\text{lb}}}{8.34 \cancel{\text{lb}}\text{/gal}} = 89.9 \text{ gals. or } 90 \text{ gals.}$$

F. Force, Pressure, Head

In order to study the forces and pressures involved in fluid flow, it is first necessary to define the terms used.

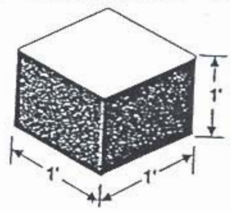
Force: The push exerted by water on any surface being used to confine it. Force is usually expressed in pounds, tons, grams, or kilograms.

Pressure: The force per unit area. Pressure can be expressed in many ways, but the most common term is pounds per square inch (psi).

Head: Vertical distance from the water surface to a reference point below the surface. Usually expressed in feet or meters.

An example should serve to illustrate these terms:

If water were poured into a one-foot cubical container, the **force** acting on the bottom of the container would be 62.4 pounds.



The **pressure** acting on the bottom would be 62.4 pounds per square foot. The area of the bottom is also 12 in. x 12 in. = 144 in^2. Therefore, the pressure may also be expressed as:

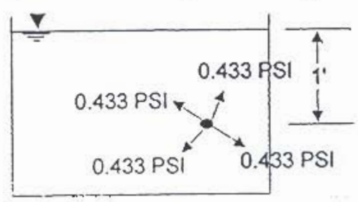
$$\text{Pressure, psi} = \frac{62.4 \text{ lb}}{\text{sq.ft.}} = \frac{62.4 \text{ lb/sq ft}}{144 \text{ sq.in./sq.ft.}}$$

$$= 0.433 \text{ lb/sq.in.}$$

$$= 0.433 \text{ psi}$$

Since the height of the container is one foot, the **head** would be one foot.

The pressure in any vessel at one foot of depth or one foot of the head is 0.433 psi acting in any direction.



If the depth of water in the previous example were increased to two feet, the pressure would be:

$$p = \frac{2\ (62.4 \text{ lb})}{144 \text{ sq.in.}} = \frac{124.8 \text{ lb}}{144 \text{ sq.in.}} = 0.866 \text{ psi}$$

Therefore, we can see that for every foot of head the pressure increases by 0.433 psi.

The pressure of a column of water 1 inch high can be computed as follows.

1 foot = 12 inches

Substituting inches for feet in the expression $\frac{0.433 \text{ lbs.}}{\text{sq.in. x ft.}}$

we have $\frac{0.433 \text{ lbs.}}{\text{sq.in. x 12 in.}}$ = 0.036 lbs. per square inch per inch.

In other words, a column of water 1 inch high produces a pressure of 0.036 pounds on each square inch of the surface on which it is resting.

Suppose we want to find the pressure of a column of mercury 1 inch high.

Mercury weighs 13.55 times that of water. Since a 1 inch column of water produces a pressure of 0.036 pounds per square inch, 1 inch of mercury = 0.036 pounds per square inch per inch x 13.55 = 0.49 pounds per square inch per inch.

In other words, a column of mercury 1 inch high produces a pressure of 0.49 pounds on each square inch of the surface on which it is resting.

G. Velocity

Velocity is defined as the change of position per unit of time or as the rate of change of position. In other words *velocity is speed.* When your car is going 80 miles per hour, the velocity of your car is 80 miles per hour. When we say the velocity of the car is 80 miles per hour we mean that at that speed the car will travel 80 miles in one hour. **Velocity of flow** of water is no different except that we express it in different terms, usually feet per minute or feet per second. To find the velocity of flow we divide the distance traveled by the time it took to travel that far. Thus, if an object in a flowing stream moves 30 feet in one minute the velocity is 30 feet per minute.

The velocity of water in a channel, pipe, or other conduit can be expressed in the same way. If the particle of water travels 600 feet in five minutes, the velocity is:

$$\text{Velocity, ft/min} = \frac{\text{distance, ft.}}{\text{time, minutes}}$$

$$= \frac{600 \text{ ft.}}{5 \text{ min.}}$$

$$= 120 \text{ ft/min}$$

If it is desired to express the velocity in feet per second, multiply by 1 min/60 seconds.

NOTE: $\frac{1 \text{ minute}}{60 \text{ seconds}}$ is like $\frac{1}{1}$ and does not change the relative value of the answer. It only changes the form of the answer.

$$\text{Velocity, ft/sec} = (\text{Velocity, ft/min})\ (1 \text{ min}/60 \text{ sec})$$

$$= \frac{120 \text{ ft}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60 \text{ sec}}$$

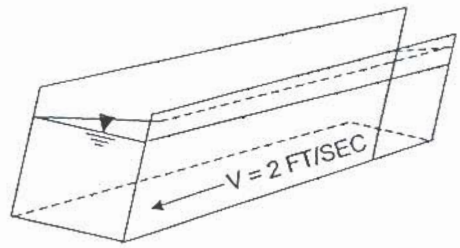
$$= \frac{120 \text{ ft}}{60 \text{ sec}}$$

$$= 2 \text{ ft/sec}$$

H. Rate of Flow (Volume-Time Relations)

We have said that velocity is the change of positions per unit of time. Applied to a stream of water it is the speed of the stream of water. We found how fast that stream of water was flowing at a certain spot, or, in other words, we found the change of position. However, we did not find out how much water was flowing in the stream past that spot in that unit of time. That is **rate of flow.** *In other words, rate of flow is the volume of water flowing past a given spot in a unit of time.* Rate of flow is usually expressed *in volume* as gallons per unit of time. Pump capacities are usually expressed in gallons per minute, and report sheets on plant operation ask for rate of flow in million gallons per day.

In the example above we found that the stream moved 120 feet in one minute or that its velocity was 2.0 ft./sec. Now let us suppose that the stream was 1 foot deep and 1 foot wide. In other words, the area of the cross section of the stream is 1 foot by 1 foot = 1 foot^2 or 1 square foot. We have also found that its velocity was 2.0 ft./sec.



To find the rate of flow of the stream we merely multiply the cross-sectional area of the stream by its velocity. In an equation it is done like this:

$$\boxed{Q = A \times V}$$

Q = rate of flow, cfs or cu.ft./sec.

V = velocity in ft./sec.

A = area, in sq.ft.

To find the rate of flow in the above channel:

$$Q = AV = 2\ \frac{\text{feet}}{\text{second}} \times 1\ \text{foot}^2 = 2\ \frac{\text{feet}}{\text{second}}$$

or

$$Q = 2\ \text{cfs.}$$

Example: Velocity in 2 foot wide channel 6 fps. If the channel is flowing 1.5 feet deep, what is the flow in the channel?

Volume - Time Measurements

Units: cubic feet per minute (ft.^{3}/min., cfm)
gallons per minute (gals./min., gpm)
cubic feet per second (ft.^{3}/sec., cfs)
million gallons per day (mgd)

Usage: 1. cubic feet/sec. - flowing rivers, canals, storm sewers
2. gallons/min. - pump capacities
3. gallons/day - water demand or water use
4. mgd - sewage flows and also as in (3) above

Conversion: cubic feet per second to mgd

1. $\frac{\text{ft}^2}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} \times \frac{60 \text{ min}}{\text{hrs}} \times \frac{24 \text{ hrs}}{\text{day}} \times 7.48 \frac{\text{gals}}{\text{ft}^3} = 646{,}000 \frac{\text{gals}}{\text{day}} = 0.646 \text{ mgd}$

2. mgd to gals/min

$$1{,}000{,}000 \frac{\text{gals.}}{\text{day}} \times \frac{\text{day}}{24 \text{ hrs.}} \times \frac{\text{hr.}}{60 \text{ min}} = 694 \text{ gals./min.}$$

That is how we find the *rate of flow of a stream*. Suppose we wanted to find the rate of flow of a pump. We could do it in the same way if we know the *area of the cross section* and the *velocity of flow* in the pipe at the pump. However, we usually have an easier way finding the capacity or rate of flow of a pump. Usually we can measure the amount of water pumped in a certain period of time and from that determine the rate of flow of the pump. Let us test the capacity of a pump by pumping into an empty rectangular tank 30 feet long by 10 feet side by 6 feet deep. We find that it takes the pump one hour and fifteen minutes to fill the tank. The pumping rate of flow of the pump is then figured as follows:

To find the volume pumped: $30 \text{ ft} \times 10 \text{ ft} \times 6 \text{ ft} \times \frac{7.48 \text{ gals}}{\text{ft.}^3} =$

first multiply the numbers $30 \times 10 \times 6 \times 7.48 = 13{,}464$

then multiply and divide the units $\text{ft} \times \text{ft} \times \text{ft} \times \frac{\text{gals}}{\text{ft}^3} =$

$$\cancel{\text{ft}^3} \times \frac{\text{gals}}{\cancel{\text{ft}^3}} = \text{gals}$$

or do both in one step $13{,}464 \cancel{\text{ft}^3} \times \frac{\text{gals}}{\cancel{\text{ft}^3}} = 13{,}464 \text{ gals}$

The volume pumped is therefore 13,464 gallons. This was pumped in one hour and 15 minutes or 75 minutes. The rate of flow or rate of pumping is therefore 13,464 gallons per 75 minutes. We have said that pump capacities were expressed in gallons per minute, so let us convert the rate of flow to gallons per minute.

$$13{,}464 \text{ gals per } 75 \text{ min} = \frac{13{,}464 \text{ gals}}{75 \text{ min}} = 179.5 \ (180) \text{ gals per min}$$

We can make this entire computation in one step:

$$30 \cancel{\text{ft}} \times 10 \cancel{\text{ft}} \times 6 \cancel{\text{ft}} \times \frac{7.48 \text{ gals}}{\cancel{\text{ft}^3} \times 75 \text{ min}} = 180 \text{ gals/min}$$

Now suppose we want to convert this to million gallons per day. We know there are 60 minutes in an hour and 24 hours in a day, or 1440 minutes in a day. If the pump will pump 180 gallons in one minute, in one day it will pump:

$$\frac{180 \text{ gals}}{\cancel{\text{min}}} \times \frac{1440 \cancel{\text{min}}}{\text{day}} = \frac{259{,}200 \text{ gals}}{\text{day}} = 259{,}200 \text{ gals per day}$$
$$= 0.2592 \text{ million gallons per day or } 0.2592 \text{ mgd}$$

Another common rate of flow used in treatment plants is gas produced or used and this is expressed in cubic feet per day.

In the first example on rate of flow, the one about the stream, we found the rate of flow of the stream by multiplying the cross-sectional area of the stream by the velocity or velocity of flow of the stream. Suppose we knew the rate of flow of the stream and want to find the velocity of flow? We would then merely work the problem in reverse or divide the rate of flow by the cross-sectional area to find its velocity. Let's try it on a little tougher problem. Suppose the rate of flow through a grit tank is 1.296 mgd (million gallons per day) and the tank is 2 feet wide and the water depth is 5 feet. We want to find the velocity of flow through the tank expressed in feet per second. Let's go.

1. Express 1.296 mgd in terms of gallons per second since we want our answer in feet per second:

   $$1296 \text{ mgd} = 1296 \text{ million gallon per day} = 1{,}296{,}000 \frac{\text{gals}}{\text{day}}$$

   1 day = 24 hours, and 1 hour = 60 minutes and 1 minute = 60 seconds; therefore, 1 day = 24 x 60 x 60 = 86,400 seconds.

   Since the rate of flow is 1,296,000 $\frac{\text{gals}}{\text{day}}$ and 1 day has 86,400 seconds, therefore:

   $$1{,}296{,}000 \frac{\text{gals}}{\text{day}} = \frac{1{,}296{,}000}{86{,}400 \text{ seconds}} = 15 \text{ gals. per sec.}$$

2. Our next step is to convert gallons to cubic feet; or let's take the 15 gals. per sec. and convert it to cubic feet per sec.

   We know that 1 cu.ft. = 7.48 gals., or 1 gal. = $\frac{1 \text{ cu.ft.}}{7.48}$

   or 0.133 cu.ft.

   $$15 \text{ gals. per sec.} = \frac{15 \text{ gals.}}{\text{sec.}} = 15 \times \frac{.134 \text{ cu.ft.}}{\text{sec.}} =$$

   2.01 cu.ft. per sec. or 2 cu. ft. per sec.

   We now have the rate of flow in cubic feet per second. Let's go to the next step.

3. Find the cross-sectional area of the grit tank. Since it is 2 feet wide and 5 feet deep, its cross-sectional area is:

   2 ft. x 5 ft. = 10 square feet or 10 ft^2

4. Now for the last step. To find the velocity we divide the rate of flow in cubic feet per second by the cross-sectional area in square feet.

Rate of flow = 2 cu.ft./sec. = 2 $\frac{ft^3}{sec}$ (step 2)

Cross-sectional area = 10 ft^2 (step 3)

therefore: $\frac{2\ ft^{\cancel{3}}}{sec. \times 10\ \cancel{ft^2}} = 0.2\ \frac{ft.}{sec} = 0.2$ ft/sec

Did canceling out these exponents throw you? (exponents are those numbers at the upper right hand corner of ft.) If they did, let's explain a little further. When we divide ft^3 by ft^2, we subtract the exponent 2 from the exponent 3 leaving an exponent of 1. (Remember that when we multiplied ft. x ft. x ft. we added exponents to get ft^3. This is just the reverse). We can prove this method of subtracting exponents as follows:

Our rate of flow was 2 $\frac{ft^3}{sec} = \frac{2\ ft \times ft \times ft}{sec}$

Our cross-sectional area was 10 ft^2 = 10 ft. x ft.

Then dividing rate of flow by area we get $\frac{2\ ft \times \cancel{ft} \times \cancel{ft}}{sec \times 10\ \cancel{ft} \times \cancel{ft}}$ = 0.2 ft/sec

Example: During 10 hours each day a station pumps with a 300 gal/min unit. What is the pump stations output in MGD?

$$300\ \frac{gal}{min} \times 60\ \frac{min}{hr} \times 10\ \frac{hr}{day} = 180{,}000\ \frac{gals}{day} = 0.18\ \text{MGD}$$

Example: Flow in a 2.5 foot wide channel is 1.4 ft. deep and measures 11.2 cfs. What is the average velocity?

Example: Flow in an 8-inch pipe is 500 GPM. What is the average velocity?

III. APPLICATIONS

A. Concentration

Concentration is the relative amount of one substance in another. In other words, when we say that Ivory soap is 99 $\frac{44}{100}$ percent pure, the concentration of the impurities in the soap is $\frac{56}{100}$ percent (100 percent minus 99 $\frac{44}{100}$ percent)

Now, instead of Ivory soap let's think of your plant effluent. You know that it is principally water and that there are also some solids carried in it. The concentration of solids in it could be expressed in percent, but since the percentage figures would be so small that they would be hard to work with, we use a different term called "parts per million." What does it mean? It means weight units per million weight units. In other words, pounds per million pounds, tons per million tons, grams per million grams, milligrams per million milligrams.

Are you acquainted with the terms "grams" and "milligrams"? They are units of weight just as ounces and pounds. One milligram is one thousandth of one gram. The term "milli" applied to any unit means one-thousandth of that unit. There are 28.35 grams in the ordinary ounce. In obtaining the concentration of a material in sewage, we must determine the small quantities of impurities in the weight units of milligrams or thousandths of grams. If we measure the milligrams of impurities in a liter of sample, we obtain the concentration in parts per million. One liter of water weighs 1000 grams or 1,000,000 milligrams. Thus, milligrams per liter is milligrams per million milligrams or parts per million parts. Note here also that a liter is 1.06 quarts and that 3.79 liters are equal to one gallon.

In plant operating reports we know the analysis in terms of parts per million and wish to use this data together with flow data to obtain the pounds of material removed. Here we wish to change parts per million removed and million gallons per day into pounds removed per day. This is done as follows:

> Suppose 150 parts per million of suspended solids are removed from sewage and the rate of sewage flow is 0.5 MGD or 500,000 gallons per day.
>
> In order to use parts per million part, we must express 500,000 gallons per day in the weight units we want for our answer, namely, pounds.
>
> One gallon weighs 8.34 lbs.
>
> $$0.5 \text{ MGD} = \frac{0.5 \text{ million } \cancel{\text{gals}}}{\text{day}} \times \frac{8.34 \text{ lbs.}}{\cancel{\text{gals}}} = \frac{4.17 \text{ million lbs.}}{\text{day}}$$

We find that 150 parts per million of suspended solids were removed, which means that 150 pounds per million pounds of sewage were removed. In order to obtain the amount of suspended solids removed per day, we must multiply this factor by the million pounds of sewage per day. Thus,

$$4.17 \frac{\text{million pounds}}{\text{day}} \times \frac{150 \text{ pounds}}{\text{million pounds}} = 626 \frac{\text{pounds}}{\text{day}} \text{ or } 626 \text{ pounds per day}$$

Likewise, concentration of sludge is expressed in percent which is parts per 100 parts, pounds per 100 pounds, tons per 100 tons, and grams per 100 grams. The use of percent is similar to the use of parts per million. Thus 15,000 gallons of liquid sludge containing 3.0 percent dry total solids contain the following quantity of dry total solids:

$$15{,}000\ \cancel{gals} \times \frac{8.34\ lbs}{\cancel{gal}} = 125{,}000\ lbs.\ liquid\ sludge,$$

and 3% dry total solids = $\frac{3\ lbs.\ dry\ total\ solids}{100\ lbs.\ liquid\ sludge}$

Then: $125{,}100\ lbs.\ \cancel{liquid\ sludge} \times \frac{3\ \cancel{lbs}.\ dry\ total\ solids}{100\ \cancel{lbs}.\ \cancel{liquid\ sludge}}$

$= 3753$ lbs. dry total solids

Units of Concentration

1. Parts Per Million (ppm)

Units: may be in pounds, ounces, liters, gallons, etc.

Examples: 1 gallon of chlorine in 1,000,000 gallons of water will give a concentration of 1 ppm.

1 gal./1,000,000 gals. = 1 ppm
or 1 oz./1,000,000 oz. = 1 ppm

Usage: If 1 lb./1,000,000 lbs. = ppm then
8.34 lbs./8,340,000 lbs. = 1 ppm
water weighs 8.34 lbs. per gallon so 8.34 lbs./MG = 1 ppm

Problem: In treating 800,000 gallons of sewage you have used 150 lbs. of a chemical. What was the dosage in ppm?

Solution: 800,000 is 0.8 of a MG and 8.34 lbs./MG = 1 ppm then 150 #/8.34 # = 18.0 ppm

$$\frac{18.0}{0.8} = 22.5\ ppm$$

Problem: Odors arising in a sewage system, especially long outfalls may be prevented by holding decomposition in check by up-sewer chlorination. Suppose it is desired to add 15 ppm of chlorine to a line handling sewage at the rate of 0.5 mgd. How much chlorine needs to be added to give 15 ppm chlorine in the line?

Solution: 8.34 lbs. chlorine added per MG = 1 ppm then

$$8.34 \times \frac{500{,}000}{1{,}000{,}000} = 4.17\ lbs./0.5\ MG = 1\ ppm$$

for 15 ppm: 4.17 lbs. x 15 = 62.4 lbs. will be used for each 0.5 mgd

for hourly rate: $\frac{62.4\ lbs.\ day}{24\ hours} = 2.6\ lbs\ \frac{cl}{hr.}$

Problem: If an Imhoff cone holds 9 ml. of solids. What are the ppm's of settleable solids?

$$\frac{9\text{ ml}}{1{,}000\text{ ml}} = \frac{9{,}000\text{ ml}}{1{,}000{,}000\text{ ml}}$$

these have the same ratios, but B is 1,000 times as large as A.

$$\frac{1\text{ ml}}{1{,}000{,}000\text{ ml}} = 1\text{ ppm so, } \frac{9{,}000}{1{,}000{,}000\text{ ml}} = 9{,}000\text{ ppm}$$

Problem: How many ppm in a 0.02% solution?

Solution: Expressing 0.02% in another way, 0.02 parts/100 parts, so $\frac{0.02}{100} = \frac{200}{1{,}000{,}000\text{ ml}}$ so answer = 200 ppm

Example: An analysis of a certain sewage shows that it contains 460 ppm of suspended solids. How many pounds of suspended solids are there in 40,000 gallons of the sewage?

Solution: We must first convert the 40,000 gallons of water to pounds. From # Appendix A we find that one gallon of water or sewage weighs 8.34 pounds. Therefore, 40,000 x 8.34 = 333,600 lbs. 460 ppm may also be written 460/1,000,000 or 0.000460. Therefore, how many pounds of total solids are there in 333,600 pounds of the mixture if it contains 460 ppm of total solids?

This may be set up on the form of a proportion --

$$\frac{?\text{ pounds}}{333{,}600\text{ pounds}} = \frac{460\text{ parts}}{1{,}000{,}000\text{ parts}}$$

Cross-multiplying, we find that it contains 153.5 pounds of total solids.

2. Grains Per Gallon

The term grains per gallon is often used in connection with sewage and water treatment, usually when referring to the chemical treatment such as, the alum dose is 1 grain per gallon. A grain is a definite unit of weight just as is the pound or ounce. From Appendix A, there are 7,000 grains in one pound. Therefore, when we say that the alum dose is 1 grain per gallon we mean that grain or 1/7,000 pound alum is added to each gallon water.

Example: A certain water plant treats 10,000,000 gallons of water on a certain day and the alum dosage is 1 grain per gallon. How many pounds of alum were used on that day?

$$\frac{1}{7{,}000} \times 10{,}000{,}000 = 1428.5\text{ pounds of alum}$$

Problem: If 600 pounds of alum per day are used for a flow of 4 mgd, what is the average dose in grains per gallon?

3. Milligrams Per Liter

Milligrams per liter (mg/l) is a unit of expression used in laboratory and scientific work to indicate very small concentrations of dissolved substances and solids, and since small amounts of chemical compounds are sometimes used in wastewater treatment processes, the term milligrams per liter is used in wastewater treatment processes, the term milligrams per liter is also common in treatment plants. It is a weight/volume relationship.

As previously discussed:

1000 liters = 1 cubic meter = 1,000,000 cubic centimeters

Therefore:

1 liter = 1000 cubic centimeters

Since one cubic centimeter of water weighs one gram,

1 liter of water = 1000 grams or 1,000,000 milligrams

$$\frac{1\ \text{milligram}}{\text{liter}} = \frac{1\ \text{milligram}}{1,000,000\ \text{milligrams}} = \frac{1\ \text{part}}{\text{million parts}} = 1\ \text{part per million ppm)}$$

Milligrams per liter and parts per million (parts) may be used interchangeably as long as the liquid density is 1.0 gm/cu cm or 62.43 lb/cu ft. A concentration of 1 milligram/liter (mg/l) or 1 ppm means that there is 1 part of substance by weight for every 1 million parts of water. A concentration of 10 mg/l would mean 10 parts of substance per million parts of water.

To get an idea of how small 1 mg/l is, divide the numerator and denominator of the fraction by 10,000. This, of course, does not change its value since 10,000 ÷ 10,000 is equal to one.

$$\frac{1\ \text{mg}}{1} = \frac{1\ \text{mg}}{1,000,000\ \text{mg}} = \frac{1/10,000\ \text{mg}}{1,000,000/10,000\ \text{mg}} = \frac{0.0001\ \text{mg}}{100\ \text{mg}} = 0.0001\%$$

Therefore, 1 mg/l is equal to one ten-thousandth of a percent, or:

1% is equal to 10,000 mg/l

To convert mg/l to %, move the decimal point four places or digits to the left.

Working problems using milligrams per liter or parts per million is a part of everyday operation in most treatment plants.

Example: A plant effluent flowing at a rate of five million pounds per day contains 15 mg/l of solids. How many pounds of solids will be discharged per day?

$$15 \text{ mg/l} = \frac{15 \text{ lbs solids}}{\text{million lbs water}}$$

Solids Discharged, lbs/day = Concentration, lbs/M lbs x Flow, lbs/day

$$= \frac{15 \text{ lbs}}{5 \cancel{\text{million lbs}}} \times \frac{5 \cancel{\text{million lbs}}}{\text{day}}$$

$$= 75 \text{ lbs/day}$$

There is one thing that is unusual about the above problem and that is that the flow is reported in pounds per day. In most treatment plants flow is reported in terms of gallons per minute or gallons per day. To convert these flow figures to weight, an additional conversion factor is needed. It has been found that one gallon of water (and wastewater, since it is almost all water) weighs 8.34 pounds. Using this factor, it is possible to convert flow in gallons per day to flow in pounds per day.

Example: A plant influent of 3.5 million gallons per day (MGD) contains 200 mg/l BOD. How many pounds of BOD enter the plant per day?

$$\text{Flow, lbs/day} = \text{Flow, } \frac{\text{M gal}}{\text{day}} \times 8.34 \frac{\text{lb}}{\text{gal}}$$

$$= \frac{3.5 \text{ million } \cancel{\text{gal}}}{\text{day}} \times \frac{8.34 \text{ lbs}}{\cancel{\text{gal}}}$$

$$= 29.19 \text{ million lbs/day}$$

BOD Loading, lbs/day = Concentration, mg/l x Flow, M lb/day

$$\text{BOD} = \frac{200 \text{ mg*}}{\cancel{\text{million}} \text{ mg}} \times \frac{29.19 \cancel{\text{million}} \text{ lbs}}{\text{day}}$$

$$= 5838 \text{ lbs/day}$$

In solving the above problem a relation was used that is most important:

$$\boxed{\text{lbs/day} = \text{Cons., mg/l} \times \text{Flow, MGD} \times 8.34 \text{ lb/gal}}$$

Example: Treated effluent is pumped to a spray disposal field by a pump that delivers 500 gallons per minute. Suspended solids in the effluent average 10 mg/l. What is the total weight of suspended solids deposited on the spray field during a 24 hour day of continuous pumping?

B. Chlorine (CL_2) Feed Rate

Milligrams per liter and parts per million may be used interchangeably. A concentration of 1 milligram/liter (mg/l) or 1 ppm means that there is 1 part of substance by weight for every 1 million parts of liquid.

Working problems using milligrams per liter or parts per million is a part of everyday operation in most wastewater treatment plants.

Formula: Cl_2 feed rate, lbs/day = dose mg/l x flow MGD x 8.34 lbs/gal

Example: The chlorine dose is 7.5 ppm. The average daily flow is 0.8 mgd. Calculate the amount of chlorine in lbs/day used.

$$\begin{aligned} Cl_2 \text{ feed rate, lbs/day} &= \text{dose mg/l} \times \text{flow MGD} \times 8.34 \text{ lbs/gal} \\ &= 7.5 \text{ mg/l} \times 0.8 \text{ MGD} \times 8.34 \text{ lbs/gal} \\ &= 6 \times 8.34 \\ &= 50.04 \text{ lbs/day} \end{aligned}$$

Example: A chlorinator is set to feed 50 pounds of chlorine per day to a flow of 0.8 MGD. What is the chlorine dose in ppm?

$$\begin{aligned} \text{Conc. or Dose} &= \frac{\text{lbs/day}}{\text{MGD} \times 8.34 \text{ lb/gal}} \\ &= \frac{50 \text{ lb/day}}{0.80 \text{ MG/day} \times 8.34 \text{ lb/gal}} \\ &= \frac{50 \text{ lb}}{6.672 \text{ M lb}} \\ &= 7.5 \text{ ppm, or } 7.5 \text{ mg/l} \end{aligned}$$

C. Chlorination

Newly constructed wells are often given an initial disinfection by using a chlorine bearing powder such as calcium hypochlorite. This powder is not pure and its strength if often stated as having a certain percentage of available chlorine.

Example A: A newly constructed well is 4 ft. in diameter and has 12 feet of water in it. To give this well an initial disinfection with 50 ppm chlorine, how much chlorine powder of 60% available chlorine do you use?

Solution: It is necessary to first determine the amount of water in the well as follows:

$$\pi R^2 \times H = 3.1416 \times 2^2 \times 12 = 150.8 \text{ cu.ft.}$$

This volume of water must next be converted to pounds of water, since ppm are usually calculated by weight. Therefore multiply by 62.4 because a cubic foot of water weighs 62.4 pounds.

150.8 x 62.4 = 9409 lbs. of water in the well

To determine the amount of chlorine bearing powder necessary to give a 1 ppm solution, divide by 1,000,000.

$$\frac{9409}{1{,}000{,}000} = 0.009409 \text{ lbs. needed to give 1 ppm.}$$

Since we desire 50 ppm, this result is multiplied by 50 to give us the amount necessary to obtain a 50 ppm solution.

50 x 0.009409 = 0.47045 lbs. of chlorine required.

However, the calcium hypochlorite that we intend to use contains only 60 % available chlorine. Therefore we must multiply the amount of chlorine required by 100/60

$0.47045 \times \underline{100} = 0.785$ of chlorine powder.

Since there are 16 ounces in one pound, we can convert this result to ounces by multiplying by 16.

0.785 x 16 = 12.6 ounces

All these steps may be set up in equation form, so that whenever a problem arises, the values may be substituted in the formula and the result calculated.

Therefore:

$$\text{Cu.ft. of water} \times \text{wt. of cu.ft. of water} \times \text{ppm} \; \frac{100}{\text{\% of available } Cl_2}$$

$$\frac{3.1416 \times 2^2 \times 12 \times 62.4 \times 50}{1{,}000{,}000} \times \frac{100}{60} = .785 \text{ pounds}$$

0.785 x 16 = 12.56 ounces

Example B: The third compartment of a 3-compartment sink holds 20 gallons of water and we want to make a 200 ppm quaternary ammonia (Q.A.) sanitizing rinse using a commercial quaternary ammonium compound of 10% strength. As we are using a liquid, the answer should be in fluid ounces so the restaurant operator will be able to measure it. (Assume that the quaternary ammonium compound weighs the same as the water)

Solution: This problem may be solved as was the previous one, since we are dealing with two liquids of the same density (water and quaternary ammonium).

Substituting in the formula described in Example A:

$$\frac{20 \times 8.34 \times 200}{1{,}000{,}000} \times \frac{100}{10} = 0.3336 \text{ lbs. of Q.A. required}$$

Since we want the answer in fluid ounces, we first convert the pounds of quaternary ammonium compound to gallons by dividing by 8.34.

$$\frac{0.3336}{8.34} = 0.04 \text{ gallons}$$

To convert gallons to fluid ounces multiply by 128 since there are 128 fluid ounces in one gallon.

$0.04 \times 128 = 5.12$ fluid ounces

Problem: If a flow is 300,000 gal/day and a dose of 2 ppm applied, how many pounds of chlorine will be used in 30 days?

Problem: If 20 pounds of chlorine per day are used for a flow of 3 mgd, what is the average rate of dosing?

D. Proportions

A problem that can be solved by the use of proportions is encountered in the checking of high-temperature short-time (HTST) pasteurizers. The speed of the pump which delivers the milk through the holding tube of a HTST pasteurizer is often set by using water and injecting a salt solution into this water and determining electrically the time necessary for the water to flow through the holding tube. As the time for milk to flow through holding tube is not necessarily the same as for water, we must determine the difference in the rate of flow of these two liquids. This can be done by determining the time necessary to fill a 40 qt. can when milk is being pumped through.

Example: A HTST pasteurizer was checked and the following information was obtained:

Time of flow of water through holding tube was 18 seconds.
Time for water to fill a 40 qt. can - 1 minute, 54 seconds.
Time for milk to fill a 40 qt. can - 1 minute, 47 seconds.

How long does it take for the milk to flow through the holding tube?

1 minute, 54 seconds = 114 seconds
1 minute, 47 seconds = 107 seconds

$$\frac{114}{107} = \frac{18}{X} \; ; \; 114X = 18 \times 107 \; ; \; 114X = 1926$$

$X = 16.8$ seconds - holding time for milk

APPENDICES

A. Constants and Conversion Factors

B. Area/Volume Formulas

C. Equation Derivations

CONSTANTS AND CONVERSION FACTORS

Constants

π equals 3.1416
Water freezes at 32°F or 0°C
Water boils at 212°F or 100°C
Body temperature is 98.6°F or 37°C

ppm is 1 part in a million parts by weight
ppm = 8.34#/million gallons

1 gallon = 231 cu. inches
1 gallon of water weighs 8.34#
1 quart = 32 fluid ounces
1 cu.ft. = 7.48 gallons (7.5)
1 cu.ft. of water weighs 62.4# app.
1 cu.ft. = 1728 cu.in.

7000 grains = 1#
1 grain/gallon = 17.1 ppm
1 grain/gallon = 142.86#/mil.gal.

1 atmosphere pressure = 14.7 #/sq.in.
1 atm. pressure = 34 ft. of water
1 foot of water = 0.433 #/sq.in.
1 #/sq.in. = 2.31 ft. of water

1 mile = 5280 feet
1 acre = 43,560 sq.ft.

1 British imperial gallon = 1.2 U.S. gallon
1 British imperial gallon weighs 10#

Diameter of pipe or cylinder in inches squared x 0.041 equals gallons per foot run

To convert Centigrade to Fahrenheit temperature multiply the centigrade reading by 9/5 and add 32

To convert Fahrenheit to Centigrade temperature subtract 32 from Fahrenheit reading and multiply by 5/9

Calorie is the amount of heat required to raise 1 gram of water through 1°C (This is a small calorie)

Large calorie is amount of heat required to raise 1 kilogram of water through 1°C (large calorie = 1000 small calorie)

Formulas

Circumference of circle = πD
Area of circle is πR^2 or .785 D^2

Metric System

Milli 1/1000 or .001
Centi 1/100 or .01
Deci 1/10 or .1
unit 1
Deka 10
Hecto 100
Kilo 1000

liter - capacit
meter - length
gram - mass

Examples

1 milliliter (ml) is 1/1000 liter
1 milligram (mg) is 1/1000 gram
1 kilogram (kg) is 1000 grams

Equivalents

-Length-

Centimeter - 0.3937
Meter - 3.28 feet
Kilometer - 0.621 miles
Inch - 2.540 centimeters (cm)
Foot - 0.305 meter (m)

-Area-

Sq. centimeter - 0.155 sq.in.
Sq. meter - 10.76 sq.ft.
Hectare - 2.47 acres
Sq. kilometer - 0.386 sq. mile
Sq. inch - 6.45 sq. centimeter
Sq. foot - 0.0929 sq. meter
Acre - 0.405 hectare
Sq. mile - 2.59 sq. kilometer

-Capacity-

Milliliter - 0.0338 U.S. liquid oz.
Liter - 1.057 U.S. liquid quarts
Liter - 0.2642 U.S. gallon
U.S. liquid oz. - 29.57 milliliter
U.S. liquid quart - 0.946 liter
U.S. gallon - 3.785 liters

-Weight-

Gram - 15.43 grains
Gram - 0.0353 Avoir. ounce
Kilogram - 2.205 Avoir. pounds
Grain - 0.0648 gram (g)
Avoir. pound - 0.4536 kilogram (kg)
Avoir. ounce - 28.35 grams (g)

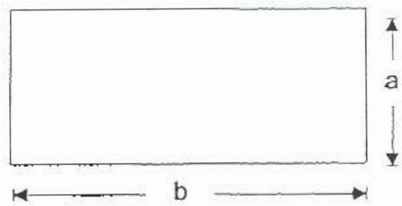
APPENDIX B

AREA/VOLUME FORMULAS

Area Formulas

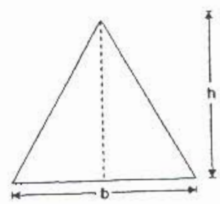
Rectangle

Area = a x b



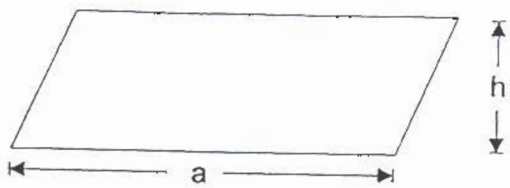
Triangle

Area = $\frac{b \times h}{2}$



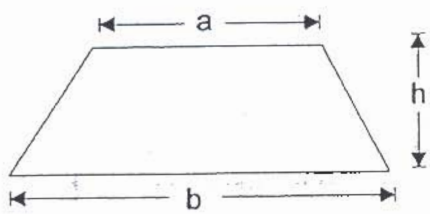
Parallelogram

Area = a x h



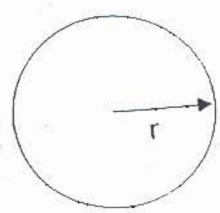
Trapezoid

Area = $\frac{h(a+b)}{2}$



Circle

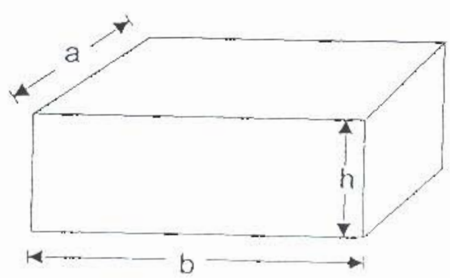
Area = πR^2 = 3.1416 x R x R



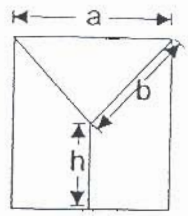
Volume Formulas

Parallelopiped

Volume = a x b x h

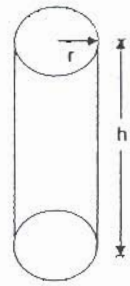


Prism



$$\text{Volume} = \frac{a \times b \times h}{2}$$

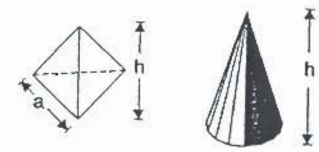
Cylinder



$$\text{Volume} = \pi \times R^2 \times h$$
$$= 3.1416 \times R \times R \times h$$

Pyramid or Cone

$$\text{Volume} = \frac{\text{area of base} \times \text{height}}{3}$$



APPENDIX C

EQUATION DERIVATIONS

I. $Q = AV$ (1)

Q = Quantity of air (cubic feet per minute) (CFM)
A = Area in square feet
V = Velocity (feet per minute) (FPM)

$V = 4005\sqrt{h_v}$ (2)

V = Velocity (FPM)
h_v = Velocity pressure expressed in inches of water

$Q = AV$ substituting equation (2) for velocity we obtain the following:

$Q = (4005\sqrt{h_v})\ (A)$ (3)

Ce = coefficient of entry for open end pipe, orifice, etc. Dimensionless. Have been worked out for various types of hoods and listed in tables. Also, obtained from

$\sqrt{h_v}$

$Ce = \sqrt{hs}$ (4)

h_v = velocity pressure - inches of water
hs = hood static pressure at hood connection to exhaust pipe, expressed in inches of water, obtained by field measurement

$\sqrt{h_v} = Ce\sqrt{hs}$ (5)

Substituting equation (5) for $\sqrt{h_v}$ in equation (3) we obtain:

$Q = (4005)*(A)*Ce\sqrt{hs}$ (6)

II. Area of Circle $= \pi R^2$ (7)

$\pi = 3.1416$
R = Radius in inches
A = Area in square inches

Also $= \frac{\pi D^2}{4}$ $\qquad D$ = diameter in inches $= 2R$ (8)

NOTE: Areas are usually referred to in square feet in this type of work. Therefore, to convert square inches to square feet we divide by 144 square inches.

Thus equations (7) and (8) become:

$$A = \frac{\pi R^2}{144} \quad (9)$$

$$A = \frac{\pi D^2}{4(144)} = \frac{\pi D^2}{576} \quad (10)$$